

Chapter 1:

- only simplify fraction vertically and diagonally
- ALWAYS FACTORISE
- when there are 3 terms in the question work on 2 first then bring back 3rd term
- when x^2 has a coefficient, after factorising make sure the coefficient would still be there
- if you want to have same denominator here

$$\frac{3x}{(x+4)^2} - \frac{1}{(x+4)} \quad \text{you cannot do this} \quad \frac{1}{x} \left(\frac{1}{x+4} \right)^2 \rightarrow \text{multiply the whole fraction by } (x+4)$$

- Improper Algebraic Fractions

- the exponent in the numerator is larger than in the denominator

$$\frac{x^4 + 5x + 8}{x - 2} \quad x^2 > x^1$$

- the exponent in the numerator is equal to the one in the denominator

$$\frac{x^2 + 5x - 4}{x^2 - 4x^2 + 7x - 3} \quad x^2 = x^2$$

To turn into mixed fractions

$$1 - \text{long division} \quad \frac{x^3 + 2x^2 + 3x - 4}{x + 1} \equiv Q(x) + \frac{\text{remainder}}{\text{divisor}}$$

$$\begin{array}{r} x^2 + x + 2 \\ x+1 \overline{) x^3 + 2x^2 + 3x - 4} \\ \underline{x^3 + x^2 + 2x} \\ x^2 + x + 2 \\ \underline{x^2 + x + 1} \\ -1 \end{array} \rightarrow x^2 + x + 2 + \frac{-1}{x+1}$$

-6

$$2 - F(x) = Q(x) \times \text{divisor} + \text{remainder}$$

$$\begin{array}{ccccc} 8x^2 + 2x^2 + 5 & \equiv & (Ax+B)(2x^2+2) & + & [Cx+D] \\ f(x) & & Q & \quad & \text{Remainder} \end{array}$$

Q Di

can switch but use what's easier where Di is known

$$\begin{array}{r} Q(x) \\ Di \overline{) f(x)} \end{array}$$

R

- with long division only divide with first term with in divisor and first time in $f(x)$
and then multiply the term you get with all of the divisor

Chapter 2:

draw sketch and see where graphs intersect
not always ++ and --

- if solving equation has 2 absolute value equations take once as (+ve +ve) and then (-ve -ve)
- solving absolute value take once as (+) and once as (-) get 2 answers
- if $|x-2| = -7$ then it has no solution
- Function $\rightarrow$ one domain (x) has one ^{image from} range (y)  one-to-one
- multiple domains (x) have one image from range (y)  many-to-one
- Not a function $\rightarrow$ One element in domain (x) has multiple images in range (y)  one-to-many
- * Vertical line test (!): passes one point $\rightarrow$ F
- passes two points $\rightarrow$ NF
- if it was a function do Horizontal line test (-): passes one point $\rightarrow$ one-to-one
- passes two points $\rightarrow$ many-to-one
- any equation with asymptotes is not a function
- $\rightarrow > \text{ or } <$
- $\rightarrow \geq \text{ or } \leq$
- if you have quadratic equation and want the inverse put it in the form of completing the square
- $a\left(\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2\right) + c$
- when asked for range and domain draw small sketch
- inverse function is reflection of original function across $y=x$
- domain of main = range of inverse
- range of main = domain of inverse
- $|f(x)| \rightarrow$ reflect any parts where $f(x) < 0$ over x-axis (no -ve y and reflect) 
- $f(|x|) \rightarrow$ erase $x \leq 0$ then reflect $x \geq 0$ over y-axis (erase -ve x then reflect +ve x) 
- Translation x axis
- $f(x+a)$ $+a \rightarrow$ left $-a \rightarrow$ right y axis
- $f(x)+a$ $+a \rightarrow$ up $-a \rightarrow$ down
- Stretch $f(ax)$ multiply x by $\frac{1}{a}$ $af(x)$ multiply y by a
- Reflection $-f(x)$ over x-axis $f(-x)$ over y-axis
- Ex: $f(x) = 5(3x-13) + 2$ with x translate ($\pm$) then stretch ($\frac{1}{5}$)
- $(0,0) \rightarrow (\frac{13}{3}, 2)$ with y stretch (a) then translate ($\pm$)
- be careful $10-11 = 01$
- $f(x) = f'(x) = x$] make one of the functions $= x$] as $f''(x)$ and $f(x)$ intersect at $y=x$
- if $g(a) = f''(a) \rightarrow f_g(a) = ff'(a) \rightarrow f_g(a) = a$
- when asked $f''(x)$ give domain even if not asked
- write $f'(x)$ not y'

Chapter 3:

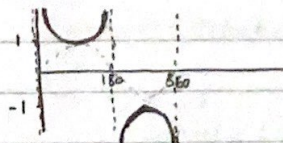
- $\sin \theta = \sin(180 - \theta)$

- don't forget to add 360 and -360 to angles you find or if 2θ its okay if its not in range still

- $\operatorname{cosec} x = \frac{1}{\sin x}$

range: $y \leq -1$ or $y \geq 1$

domain: $0 < x < 180$ and $180 < x < 360$



- $\sec x = \frac{1}{\cos x}$

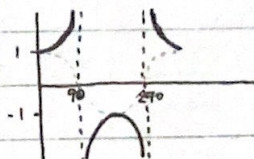
range: $y \leq -1$ or $y \geq 1$

$0 \leq x < 90$

domain: $0 \leq x < 90$ and $90 < x < 270$

and

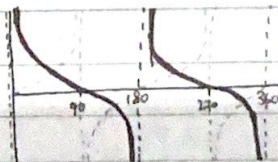
$270 < x \leq 360$



- $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$

range: $y \in \mathbb{R}$

domain: $0 < x < 180$ and $180 < x < 360$



- $\sin^2 \theta + \cos^2 \theta = 1$

$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{1}{\cot \theta}$

$\cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$

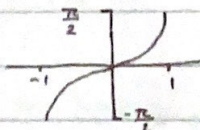
$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta} \rightarrow \tan^2 \theta + 1 = \sec^2 \theta$

$\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta} \rightarrow \cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$

$\arcsin x = \sin^{-1} x$

range: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

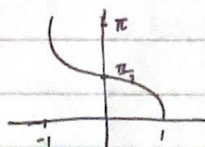
domain: $-1 \leq x \leq 1$



$\arccos x = \cos^{-1} x$

range: $0 \leq y \leq \pi$

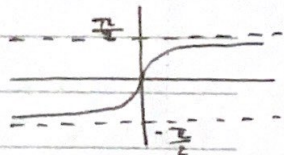
domain: $-1 \leq x \leq 1$



$\arctan x = \tan^{-1} x$

range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

domain: $x \in \mathbb{R}$



Chapter 4:

$$\sin(A+B) \equiv \sin A \cos B + \sin B \cos A$$

$$\sin(A-B) \equiv \sin A \cos B - \sin B \cos A$$

$$\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) \equiv \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Proof for $\tan(A+B)$:

$$\begin{aligned} \tan(A+B) &= \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B} \quad \text{divide numerator and denominator by } \cos A \cos B \\ &= \frac{\frac{\sin A \cos B}{\cos A \cos B} + \frac{\sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} - \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A + \tan B}{1 - \tan A \tan B} \end{aligned}$$

Proof for $\tan(A-B)$:

$$\begin{aligned} \tan(A-B) &= \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A \cos B - \sin B \cos A}{\cos A \cos B + \sin A \sin B} \quad \text{divide numerator and denominator by } \cos A \cos B \\ &= \frac{\frac{\sin A \cos B}{\cos A \cos B} - \frac{\sin B \cos A}{\cos A \cos B}}{\frac{\cos A \cos B}{\cos A \cos B} + \frac{\sin A \sin B}{\cos A \cos B}} = \frac{\tan A - \tan B}{1 + \tan A \tan B} \end{aligned}$$

$$\sin 2A = \sin(A+A) = \sin A \cos A + \sin A \cos A = 2 \sin A \cos A \quad \left. \begin{aligned} \sin 2A &\equiv 2 \sin A \cos A \\ \tan 2A &\equiv \frac{2 \tan A}{1 + \tan^2 A} \end{aligned} \right\}$$

$$\tan 2A = \tan(A+A) = \frac{\tan A + \tan A}{1 + \tan A \tan A} = \frac{2 \tan A}{1 + \tan^2 A}$$

$$\cos 2A = \cos(A+A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A \quad \left. \begin{aligned} \cos 2A &\equiv \cos^2 A - \sin^2 A \\ \cos 2A &\equiv 2 \cos^2 A - 1 \\ \cos 2A &\equiv 1 - 2 \sin^2 A \end{aligned} \right\}$$

$$\cos^2 A - (1 - \cos^2 A) = 2 \cos^2 A - 1$$

$$(1 - \sin^2 A) - \sin^2 A = 1 - 2 \sin^2 A$$

• never divide by any $\sin \theta / \cos \theta$ anything, that way you would lose an answer

- focus if they want min or max if the $\sin \theta$ is numerator or denominator

↳ if they want max and $\sin \theta$ in denominator then $\sin \theta$ min

$$a \sin \theta + b \cos \theta \rightarrow R \sin(\theta + \alpha)$$

$$a \sin \theta - b \cos \theta \rightarrow R \sin(\theta - \alpha)$$

$$a \cos \theta + b \sin \theta \rightarrow R \cos(\theta - \alpha)$$

$$a \cos \theta - b \sin \theta \rightarrow R \cos(\theta + \alpha)$$

same thing depends

on what question wants

$$3 \sin \theta + 4 \cos \theta:$$

$$\text{- Expand } R \sin(\theta + \alpha) = R[\sin \theta \cos \alpha + \sin \alpha \cos \theta] = R \sin \theta \cos \alpha + R \sin \alpha \cos \theta$$

$$R \sin \theta \cos \alpha + R \sin \alpha \cos \theta = 3 \sin \theta + 4 \cos \theta$$

$$R \sin \theta \cos \alpha = 3 \sin \theta \rightarrow R \cos \alpha = 3$$

$$R \sin \alpha \cos \theta = 4 \cos \theta \rightarrow R \sin \alpha = 4$$

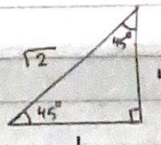
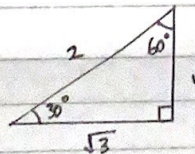
} be careful if smth is -ve

$$\frac{R \sin \alpha}{R \cos \alpha} = \frac{4}{3} = \tan \alpha = \frac{4}{3}$$

$$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$$

} should always be +ve and inside range

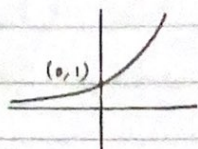
$$R = \sqrt{a^2 + b^2}$$



Chapter 5:

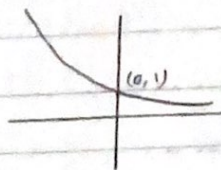
$$y = a^x$$

$y=0$ asymptote



$$a > 1$$

increasing function



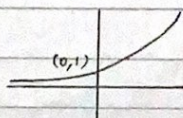
$$0 < a < 1$$

decreasing function

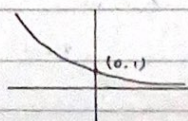
- a can't be -ve because its neither increasing or decreasing

- a can't equal 1 bc $y=1^x$ then all $y=1$ so becomes straight line

- the smaller the base (a) the closer the curve is to x-axis

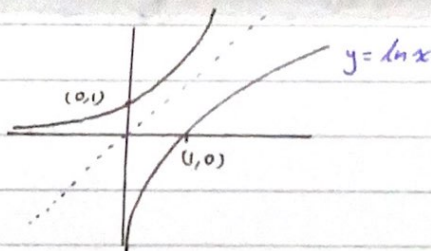


$$y = e^x$$



$$y = e^{-x}$$

$y = \ln x$ ← inverse of $y = e^x$



$$\ln e^x = e^{\ln x} = x$$

$\ln$ has same rules as $\log$

$$f(x) = Ae^{Bx} + C \text{ asymptote}$$

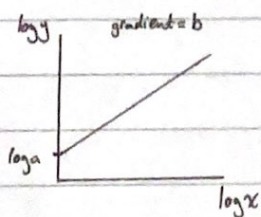
$$y = ax^b \text{ (curve)}$$

$$\log y = \log ax^b$$

$$\log y = \log a + b \log x$$

$$\boxed{\log y} = \boxed{b \log x} + \boxed{\log a}$$

$$y = mx + c$$



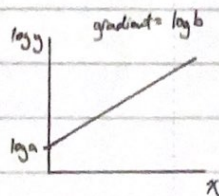
$$y = ab^x \text{ (exponential)}$$

$$\log y = \log ab^x$$

$$\log y = x / \log b + \log a$$

$$\boxed{\log y} = \boxed{(\log b)} \boxed{x} + \boxed{\log a}$$

$$y = mx + c$$



Chapter 6:

$$f(x) = \sin kx$$

$$f'(x) = k \cos kx$$

$$f(x) = \cos kx$$

$$f'(x) = -k \sin kx$$

$$f(x) = e^{kx}$$

$$f'(x) = k e^{kx}$$

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x}$$

$$f(x) = a^x$$

$$f'(x) = \ln(a) \cdot a^x$$

$$f(x) = a^{kx}$$

$$f'(x) = k \cdot \ln(a) \cdot a^{kx}$$

don't forget
 $f(x) = \ln e$
 $f'(x) = 0$

- Proof for $y = a^x \rightarrow \frac{dy}{dx} = \ln(a) \cdot a^x$

$$y = a^x \rightarrow y = e^{\ln a^x} \rightarrow y = e^{x(\ln a)}$$

$$\frac{dy}{dx} = \ln a \cdot e^{x \ln a} \rightarrow \frac{dy}{dx} = \ln a \cdot e^{\ln a^x}$$

$$\frac{dy}{dx} = \ln a \cdot a^x$$

- Proof for $y = a^{kx} \rightarrow \frac{dy}{dx} = k \ln a \cdot a^{kx}$

$$y = a^{kx} \rightarrow y = e^{\ln a^{kx}} \rightarrow y = e^{x(k \ln a)}$$

$$\frac{dy}{dx} = k \ln a \cdot e^{x k \ln a} \rightarrow \frac{dy}{dx} = k \ln a \cdot e^{\ln a^{kx}}$$

$$\frac{dy}{dx} = k \ln a \cdot a^{kx}$$

$$f(x) = e^{x^2-1}$$

$$f'(x) = 2x e^{x^2-1} \quad \left[\text{derivative of power} \cdot e^{\text{original power}} \right]$$

$$f(x) = \ln(x^2)$$

$$f'(x) = \frac{2x}{x^2} = \frac{2}{x} \quad \left[\text{derive inside } \ln \text{ inside } \ln \right]$$

- Function rotation

$$f(x)^n$$

$$f'(x) = n f(x)^{n-1} f'(x)$$

$$f(x) = (\sin x)^3$$

$$f'(x) = 3 \sin^2 x \cos x$$

differentiation using first principle for $\cos x$ and $\sin x$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

using $\sin(A+B)$
 $\cos(A+B)$
 given that $\frac{\sin h}{h} \rightarrow 1$
 $\frac{(\cos h - 1)}{h} \rightarrow 0$

$$f(x) = \sin x$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \sin x \frac{(\cos h - 1)}{h} + \frac{\sin h}{h} \cos x$$

$$\sin x(0) + (1) \cos x = \cos x$$

$$f(x) = \sin x$$

$$f'(x) = \cos x$$

- Chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = f(g(x))$$

$$\frac{dy}{dx} = f'(g(x)) g'(x)$$

$$y = (1+2x)^5$$

$$\text{let } u = 1+2x$$

$$y = u^5$$

$$\frac{du}{dx} = 2$$

$$\frac{dy}{du} = 5u^4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 5u^4 \times 2 = 10(1+2x)^4$$

$$- f(x) = \sin(ax+b)$$

$$f'(x) = a \cos(ax+b)$$

$$- f(x) = \cos(ax+b)$$

$$f'(x) = -\sin(ax+b)$$

$$- \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

- Product rule:

$$y = \overset{u}{f(x)} \overset{v}{g(x)}$$

$$\frac{dy}{dx} = f'(x)g(x) + f(x)g'(x)$$

$$\frac{dy}{dx} = vu' + v'u$$

- Quotient rule

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{vu' - v'u}{v^2}$$

- point of inflection $\rightarrow$

$$\frac{d^2y}{dx^2} = 0$$

$f(x)$

$$\begin{aligned} \sin kx \\ \cos kx \\ \tan kx \end{aligned}$$

$$\cot kx$$

$$\sec kx$$

$$\operatorname{cosec} kx$$

$$\arcsin kx$$

$$\arccos kx$$

$$\arctan kx$$

$$\operatorname{arccot} kx$$

$$\operatorname{arcsec} kx$$

$$\operatorname{arccosec} kx$$

 $f'(x)$

$$\begin{aligned} k \cos kx \\ -k \sin kx \\ k \sec^2 kx \end{aligned}$$

$$-k \operatorname{cosec}^2 kx$$

$$k \tan kx \sec kx$$

$$-k \cotan kx \operatorname{cosec} kx$$

$$\frac{k}{\sqrt{1-(kx)^2}} \quad \leftarrow \text{derivative of inside}$$

$$-\frac{k}{\sqrt{1-(kx)^2}}$$

$$\frac{k}{1+(kx)^2}$$

$$-\frac{k}{1+(kx)^2}$$

$$\frac{kx(kx)^2 - 1}{k}$$

$$-\frac{kx \sqrt{(kx)^2 - 1}}{k}$$

Proof for each one:

$$y = \tan x = \frac{\sin x}{\cos x}$$

$$\frac{dy}{dx} = \frac{u'v - v'u}{v^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$y = \tan x$$

$$u = \sin x$$

$$v = \cos x$$

$$u' = \cos x$$

$$v' = -\sin x$$

$$\frac{dy}{dx} = \sec^2 x$$

$$y = \cot x = \frac{\cos x}{\sin x}$$

$$\frac{dy}{dx} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\operatorname{cosec}^2 x$$

$$y = \cot x$$

$$u = \cos x$$

$$v = \sin x$$

$$u' = -\sin x$$

$$v' = \cos x$$

$$\frac{dy}{dx} = -\operatorname{cosec}^2 x$$

$$y = \sec x = \frac{1}{\cos x} = (\cos x)^{-1}$$

$$\frac{dy}{dx} = -1(\cos x)^{-2}(-\sin x) = \frac{\sin x}{\cos^2 x} = \tan x \sec x$$

$$y = \sec x$$

$$\frac{dy}{dx} = \tan x \sec x$$

$$y = \operatorname{cosec} x = \frac{1}{\sin x} = (\sin x)^{-1}$$

$$\frac{dy}{dx} = -1(\sin x)^{-2}(\cos x) = -\frac{\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \operatorname{cosec} x$$

$$y = \operatorname{cosec} x$$

$$\frac{dy}{dx} = -\cot x \operatorname{cosec} x$$

$$y = \arcsin x \rightarrow \sin y = x$$

$$\frac{dx}{dy} = \cos y \rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$y = \arcsin x$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\cos y = \sqrt{1 - \sin^2 y}$$

$$\cos y = \sqrt{1 - x^2}$$

$$y = \arccos x \rightarrow \cos y = x$$

$$\frac{dx}{dy} = -\sin y \rightarrow \frac{dy}{dx} = -\frac{1}{\sin y}$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$y = \arccos x$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\sin^2 y + \cos^2 y = 1$$

$$\sin y = \sqrt{1 - \cos^2 y}$$

$$\sin y = \sqrt{1 - x^2}$$

$$y = \arctan x \rightarrow \tan y = x$$

$$\frac{dx}{dy} = \sec^2 y \rightarrow \frac{dy}{dx} = \frac{1}{\sec^2 y}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$y = \arctan x$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\sec^2 y = 1 + \tan^2 x$$

$$\sec^2 y = 1 + x^2$$

$$y = \operatorname{arccot} x \rightarrow \cot y = x$$

$$\frac{dx}{dy} = -\operatorname{cosec}^2 x \rightarrow \frac{dy}{dx} = -\frac{1}{\operatorname{cosec}^2 x}$$

$$\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$y = \operatorname{arccot} x$$

$$\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$\operatorname{cosec}^2 y = 1 + \cot^2 x$$

$$\operatorname{cosec}^2 y = 1 + x^2$$

$$y = \operatorname{arcsec} x \rightarrow \sec y = x$$

$$\frac{dx}{dy} = \sec y \tan y \rightarrow \frac{dy}{dx} = \frac{1}{\sec y \tan y}$$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

$$y = \operatorname{arcsec} x$$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

$$\sec^2 y = \tan^2 y + 1$$

$$\tan y = \sqrt{\sec^2 y - 1}$$

$$\tan y = \sqrt{x^2 - 1}$$

$$y = \operatorname{arccosec} x \rightarrow \operatorname{cosec} y = x$$

$$\frac{dx}{dy} = -\operatorname{cosec} y \cot y \rightarrow \frac{dy}{dx} = -\frac{1}{\operatorname{cosec} y \cot y}$$

$$\frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$$

$$y = \operatorname{arccosec} x$$

$$\frac{dy}{dx} = -\frac{1}{x\sqrt{x^2-1}}$$

$$\operatorname{cosec}^2 y = \cot^2 y + 1$$

$$\cot y = \sqrt{\operatorname{cosec}^2 y - 1}$$

$$\cot y = \sqrt{x^2 - 1}$$

Chapter 7:

$$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + c$$

$$\int e^x \cdot dx = e^x + c$$

$$\int \frac{1}{x} \cdot dx = \int x^{-1} \cdot dx = \ln|x| + c$$

$$\int \sin x \cdot dx = -\cos x + c$$

$$\int \cos x \cdot dx = \sin x + c$$

$$\int \sec^2 x \cdot dx = \tan x + c$$

$$\int \operatorname{cosec}^2 x \cdot dx = -\cot x + c$$

$$\int \sec x \tan x \cdot dx = \sec x + c$$

$$\int \operatorname{cosec} x \cot x \cdot dx = -\operatorname{cosec} x + c$$

when you have limits with integration always work in radians

only for linear

$$\int \cos(2x-1) \cdot dx \quad \begin{array}{l} \frac{1}{\text{derivative}} = \frac{1}{2} \quad \text{which would mean} \\ \text{coefficient of } x \quad \text{be this is only for linear} \end{array} \rightarrow \frac{1}{2} \sin(2x-1) + c$$

$$\int \frac{1}{2x+1} \cdot dx \rightarrow \frac{1}{2} \ln|2x+1| + c$$

this isn't linear so $\int (3x-1)^3 \cdot dx \xrightarrow{\text{act like}} \int u^3 \cdot dx = \frac{u^4}{4}$

$$\left(\frac{1}{3} \times \frac{(3x-1)^4}{4} + c = \frac{(3x-1)^4}{12} + c \right)$$

use trigonometric identities

$$\sin 2x \equiv 2 \cos x \sin x \rightarrow \cos x \sin x = \frac{1}{2} \sin 2x$$

$$\cos 2x \equiv 2 \cos^2 x - 1 \rightarrow \cos^2 x = \frac{\cos 2x + 1}{2}$$

$$\cos 2x \equiv 1 - 2 \sin^2 x \rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\sec^2 x \equiv \tan^2 x + 1 \rightarrow \tan^2 x = \sec^2 x - 1$$

$$\operatorname{cosec}^2 x \equiv \cot^2 x + 1 \rightarrow \cot^2 x = \operatorname{cosec}^2 x - 1$$

with integration there can't be multiplication

you can take multiplication with numbers outside $\int \sin 2x \cos 2x = \int \frac{1}{2} \sin 4x = \frac{1}{2} \int \sin 4x$
 $= \frac{1}{2} [-\frac{1}{4} \cos 4x]$

Reverse chain rule

case ①

$$\int k \frac{f'(x)}{f(x)} \cdot dx$$

let $y = \ln|f(x)|$ find $\frac{dy}{dx}$ to check
and then adjust any constants

Ex $\int \frac{e^{2x}}{e^{2x}+1} \cdot dx$

$$y = \ln|e^{2x}+1|$$

$$\frac{dy}{dx} = \frac{2e^{2x}}{e^{2x}+1}$$

↳ adjust $y = \frac{1}{2} \ln|e^{2x}+1| + c$

case ②

$$\int k f'(x) \cdot [f(x)]^n \cdot dx$$

let $y = [f(x)]^{n+1}$ find $\frac{dy}{dx}$ to check
and then adjust any constant

Ex $\int \frac{\sin 2x}{(3+\cos 2x)^3} \cdot dx = \int \sin 2x (3+\cos 2x)^{-3}$

$$y = (3+\cos 2x)^{-2}$$

$$\frac{dy}{dx} = -2(3+\cos 2x)^{-3}(-2\sin 2x) = 4\sin 2x (3+\cos 2x)^{-3}$$

$$\rightarrow y = \frac{1}{4} \sin 2x (3+\cos 2x)^{-3} + c$$

Exception:

in case ②: e

- you don't do +1 to the power you take e^x as $f(x)$ and find

$$\frac{dy}{dx} \text{ for } e^x \text{ alone then act as normal} \rightarrow \int \sec^2 x e^{4 \tan x} \rightarrow y = e^{4 \tan x}$$
$$\frac{dy}{dx} = 4 \sec^2 x e^{4 \tan x}$$

$$\rightarrow y = \frac{1}{4} e^{4 \tan x}$$

Chapter 8:

Locating roots

$$f(x_1) = +ve > 0$$

$$f(x_2) = -ve < 0$$

sign change implies root

Locating turning point

$$f'(x_1) = +ve > 0$$

$$f'(x_2) = -ve < 0$$

sign change implies slope changes from dec

switch depend on x_1

Ex. $f'(4) = -ve$ $f'(10) = +ve$

dec to inc

$f'(1) = +ve$ $f'(7) = -ve$

inc to dec

← to inc over interval, which implies turning point

When asked to show that for example $x = 1.441$ is a root (or t.p.)

add dp to $x \rightarrow 1.4410$ find upper and lower bound

$$1.4405$$

$$1.4415$$

one should be +ve and the other -ve so: (you write answers and < 0 or > 0)

sign change implies x lies in the range $1.4405 < x < 1.4415$ so

$x = 1.441$ correct to 3 decimal places

Iterative method

$$f(x) = 0$$

'question tells you how to rearrange formula'

Ex: $x^2 - x + 1 = 0$

ex. ~~x^2~~ is subject $x = 1 - \frac{1}{x}$

$$x^2 = x - 1$$

$$x = 1 - \frac{1}{x}$$

$$\left[x_{n+1} = 1 - \frac{1}{x_n} \right]$$

question will give you first x like x_0 then
will either tell you what x 's to find or until
the answer is constant to 3 dp

* any answer you get from any rearrangement of the equation is
an answer to the original equation

$$\int (3x+1)^4 = u^4 \rightarrow \frac{u^5}{5}$$

$$\frac{1}{3} \times \frac{(3x+1)^5}{5}$$

$$\cos^2 x + \sin^2 x + 2\cos x \sin x = (\cos x + \sin x)^2$$

$$1 - \sin^2 x = (1 + \sin x)(1 - \sin x)$$